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Mathematics News Letter

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Published under the auspices of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

This journal is dedicated to mathematics in general, to the following causes in particular (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of Mathematical Association of America and National Council of Teachers of Mathematics projects.

VOL. 6 BATON ROUGE, LA., SEPTEMBER, 1931 No. 1

A JOINT APPEAL

To the Teachers of Mathematics in Louisiana and Mississippi:

For five years the Mathematics News Letter, published at Baton Rouge, Louisiana, in eight or nine issues each school year, has been "devoted purely and simply to the cause of mathematics and its teaching, particularly in the states of Mississippi and Louisiana." The great value of the publication is recognized by its regular subscribers, including a goodly number not residing within the Louisiana-Mississippi area.

The purposes and the policies and the standards of the magazine have attracted attention and have provoked very favorable comment among leading teachers of mathematics from the elementary school to the university throughout the country. These leaders recognize and appreciate the fact that the News Letter is doing a unique work among the mathematical publications of the country.

Because of this recognition and appreciation, the two organizations known as "The Mathematical Association of America" and "The National Council of Teachers of Mathematics" have agreed, through their officers, to assist in financing a campaign to secure a much larger number of subscribers for the News Letter. These officers join heartily with us in "urging the teachers of mathematics to rally to the support of the News Letter, believing that it is a very profitable

journal for all who are interested in the improvement of mathematical teaching."

In view of the past success and the promising future of the News Letter, it would be a travesty upon the teachers of mathematics within our area if they should fail to support the journal financially and otherwise so that it may increase the usefulness of its service to mathematical teaching.

But the financial status of the publication demands immediate action upon the part of its friends and well-wishers.

Let all of us work diligently and unceasingly to secure additional subscribers for the magazine and also to increase the number of contributors of articles to the publication. In the respective counties and parishes of the two states interested readers and friends of the journal should strive to secure the largest possible clubs of subscribers. Let us wipe out the existing financial deficit and make glad the worthy and capable members of the editorial staff with the assurance that their constructive and persevering labors have yielded rich returns in the building of a really professional attitude among our teachers of mathematics.

Let us make a genuine success of our campaign for a better and more widely read Mathematics News Letter, which in turn will make better teachers of mathematics in Louisiana and Mississippi and elsewhere.

A. C. MADDOX, Natchitoches, La.
Chairman of the Louisiana-Mississippi Section
of the Mathematical Association of America.

J. T. HARWELL, Shreveport, La.,
Chairman of the Louisiana-Mississippi Branch
of the National Council of Teachers of Mathematics.

September 10, 1931.

USING THE NEWS LETTER

We have had editorials in former issues on the plan and program of the News Letter. It is well known that the Letter is the official organ of the Sections of the Association and Council. Under this provision we expect announcements of meetings, programs, and other matters of general interest. Probably the most important function of the News Letter is to provide a medium through which teachers

and students in our schools and colleges may write on topics which concern high school and college mathematics. This is an opportunity for teachers with ideas and students who are ambitious to learn. These aims will fail if the News Letter fails to find a place in the reading list of a representative number of our schools and teachers. A letter is being circulated urging teachers to place the News Letter in the library for the use of students and subscribe for a personal copy. In this way we can use the bulletin for the inspiration of high school and college students as well as for our own personal interests. To this end we should see that a suitable variety of subjects are discussed in the issues of the News Letter. We should cooperate in suggesting topics for papers. Suppose we should begin with the following and lengthen the list as fast as possible.

1. Problem types in algebra.
2. Practical constructions for practice in geometry.
3. Field exercises for students of trigonometry.
4. Amusing recreations in arithmetical numbers.
5. Determinants in elementary mathematics.
6. Objectives in high school geometry.
7. The graph as a practical device.
8. Averages and how to use them.
9. On computing a table of logarithms.
10. The principle and use of the slide rule.
11. Use of types of coordinates.
12. Graphs in the third dimension.
13. Problems proposed for solution.
14. When can numerical calculus be introduced?—C. D. S.

SOME VALUES OF THE STUDY OF MATHEMATICS

By LAWRENCE E. HEGGINS
New Orleans, La.

Mathematics occupies an unusual place in the program of studies in our secondary schools. With the exception of a few special type schools, practically every secondary school is the United States includes Algebra and Geometry in its curriculum, and, in most cases, these subjects are required of the pupils for graduation. Hence we see that a large proportion of pupils in a secondary school are always engaged in the study of mathematics; few pupils pass through the first

part of their secondary school education without some contact with the subject, and a large proportion of the total time devoted to secondary school education is occupied in the study of mathematics. "Where Algebra is studied five periods a week for one year and Geometry for a like amount of time, about one-eighth of the total time of a pupil who remains in school four years is devoted to mathematics. If he remains one or two years, about one-fourth of his total time is devoted to mathematics."[1]

What is the purpose of devoting so much time to mathematics? and what are some of its values? we may well ask ourselves. So I shall endeavor, in a small way, to show some of the values of mathematics and to justify, on the basis of its values, the time given to its study.

"For purposes of analysis, we may consider the values commonly claimed for the study of mathematics under two general headings:

I. Those values which arise from the relatively direct and specific use of mathematics, and

II. Those values which may arise indirectly through the development of mathematical concepts or through transfer of improved efficiency.

These grouped values, may in turn, be subdivided into, under I,

(a) The values of mathematics as measured by the directly practical application of its principles and processes to those affairs of life common to most people whatever be their vocations;

(b) The values of mathematics as measured by the directly practical application of its principles and processes in special professions or special parts of certain vocations.

(c) The values of mathematics as measured by the direct application of its principles and processes to other sciences.

Then under II, we have

(a) Values claimed to arise from the study of mathematics as measured by the development of generally valuable concepts of number and space relations, together with the development of certain mathematical thought modes.

(b) Values claimed to arise from the study of mathematics as measured by the transfer or spread to other fields of improved efficiency."[2]

[1]Principles of Secondary Education—A. Inglis; pps. 483-484.
[2]Principles of Secondary Education—A. Inglis; pps. 485-486.

Allan D. Campbell of Syracuse University divides the values of mathematics into five classes, namely, "the practical values, the mental values, the moral values, the spiritual values, and the aesthetic values"[3] Under practical values he shows us how the World War made extensive use of mathematics, and how many of our leading commercial firms employ mathematicians for research work in problems of mechanics, electricity, statistics, etc.

Under mental values, Mr. Campbell shows us that mathematics, because it is pure logic, stands out as a fine subject with which to train the mind, besides we are working along the road of creative logic and reasoning and discovery.

Under moral values, he has that mathematical studies are free from all prejudices, passions, sentiments, and feelings; and in their pursuit, we come as close to the perfectly moral action as we ever do. Mr. Campbell says, "We must attack a mathematical problem with patience, modesty, love for truth, absolute surrender to the laws of the subject, and absolute mental and moral candor and honesty." In many subjects we let our prejudices control and color our opinions and views, but in mathematics, our guiding reasons and principles are all open and above board.

The above mentioned abiding truth and permanent character of mathematics constitute one of the chief spiritual values of its study. Regardless of the ever shifting standards of morality, the changes in religious ideas, changes in governments, etc., mathematics ever remains upon the same, never changing foundation. It is in this feeling of security and permanence that many people find "a footing on the slippery ledges that lead to character and attainment in life."

Lastly, Mr. Campbell says that every mathematician loves his subject. Mathematics and music and poetry are all combined in a mathematician. It is mathematics as an art and not as an industry that draws the mathematician.

On the other hand, we read in Mills and Mills "Teaching of High School Subjects," that "the educational values of mathematics are primarily disciplinary and cultural. Possibly no other subject in the secondary school curriculum is better fitted in its very nature for the development of valuable mental habits. Mathematical study is progressive, advancing by steady, even graduations, producing breadth and depth of mind. It trains the mind to exact and progressive thinking, to adequacy of conception and precision of expression, to

[3]The Mathematics Teacher—January, 1931, (Vol. XXIV, No. 1) p. 46.

energy of attention, to clearness of inner vision, to the perception of necessary truths.[4] It quickens the scientific conscience, and is, in itself, a school of scientific thinking, terminology, and expression."[5]

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5. Supervised Study in Mathematics and Science—*S. Clayton Summer.*

STANDARD TESTS

By SOPHIE SCHMIDT
T. J. Semmes School, New Orleans, La.

It has been stated that "one of the best ways to clear up notions of what the functions are which schools should develop and improve is to get measures of them." Whenever any measurement is to be made there are several types of elements that enter into the situation Some of them are (1) the thing measured; (2) the *norm* or *degree of ability* which a pupil of a given grade or age should possess; (3) the method by which measurement is made; (4) and the condition under which such measurement is made. By far, the most important of these elements are the thing measured and the ability which a pupil of a given grade or age should possess. In addition to this, consideration should always be given to the fact that there are marked and distinct individual differences among pupils.

The best tools for carrying out a measurement program are standardized tests. Such tests, or measures for comparison as they may be called, are of several different types. We have intelligence tests and achievement tests. Among the latter are tests for measuring accuracy in computation alone, tests for measuring both speed and accuracy, as well as tests for measuring ability in reasoning. Besides finding place in the group of tests for measuring degree of proficiency attained by pupils, the standardized tests also fall into certain classes in accordance with the use to be made of results shown by them. One of these is the diagnostic test which shows the particular difficulty

[4]The Studies of the Secondary School—C. DeGarmo; p. 65.
[5]Teaching of High School Subjects—Mills & Mills; p. 236.

which confronted the pupil or the type of error made. This type of test is of vast importance, since it not only diagnoses the individual and class difficulties, but it can be used as an incentive to improvement on the part of pupils as well as serve as guiding star to the teacher in planning remedial instruction.

That such tests or measurements are of inestimable value in improving instruction has been shown by a recent experimental study in diagnostic and remedial work carried on by the Bureau of Reference, Research and Statistics, of New York City. At the beginning of the school term a group of 6A pupils were selected as subjects for an experiment which had for its purpose the development of a method of diagnosing difficulties, the planning of a program of remedial work, and the using of standard tests as a regular part of classroom routine.

First of all, ability was tested for. The "National Haggerty D2" test showed the group to be somewhat above the average in intelligence. Then the Woody-McCall and the Courtis Standard Research, Series B tests, were administered and these showed the group to be somewhat above the average in achievement, but wide variability in scores showed lack of uniformity in ability to handle arithmetic fundamentals. Monroe's Diagnostic Tests were administered and results showed particular deficiencies in long division and fractions.

The status of the group having been partly determined, the research workers continued in painstaking manner to diagnose individual difficulties. They administered the Wisconsin Inventory Test to the poorest pupils. Individual records were kept. Test papers, oral examinations and observations of pupils at work gave definite data on failures of individual pupils.

All this pointed the way to remedial instruction. Class and individual weaknesses were strengthened by specific instruction. Drill on the operations where deficiencies had been shown was provided by the use of "The Courtis Standard Practice Tests." The pupils were helped to realize their own deficiencies and inspired with a desire to improve their work. Each child kept a graphical record of his own achievement and the teacher made similar records of class achievement.

A graph is such a vivid representation of progress, lack of progress, or of retrogression that it is the best spur to effort that can be used and it is to be noted as a signal triumph scored when the use of

the graph has inspired the pupil with a desire to improve his work which the test proved below standard.

At the end of the term, tests in fundamentals were again given to this group to measure improvement and to compare with other 6A groups in the school. Reasoning tests were also given to determine the group's standing in problem solving "since greater emphasis had been given to fundamentals during the term." It was found that there had been much gain both in work accomplished and in habits of work. This held both for the class and for the individual members.

Among the gains found were the following:

(1) The teacher's attitude toward failures.
(2) Better understanding of the mental processes by the pupils.
(3) Increased interest in the fundamentals on the part of the pupils.
(4) The pupils' attitude toward their own deficiencies.
(5) Training in pupils' point of view toward marks and toward improvements.

Some of the conclusions that the investigators reached were (1) that standardized tests may be successfully used as part of classroom routine; (2) that standardized tests may be used instead of some of the teacher's own weekly or monthly tests. The experiment proved conclusively that "teachers should learn to use standardized tests as part of their regular work and they should not consider them as some extra activity outside their experience." The reason is that the value of standardized tests is based on the fact that they are scientifically devised and as a rule are vastly superior to those devised by the individual teacher.

The experiment referred to leads us to feel that we too may carry on such a campaign with our own classes and in consequence we may lift out of "The Slough of Despond" many pupils who otherwise would be numbered among that unhappy band called "Failures."

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ON THE LAW OF GRAVITATION

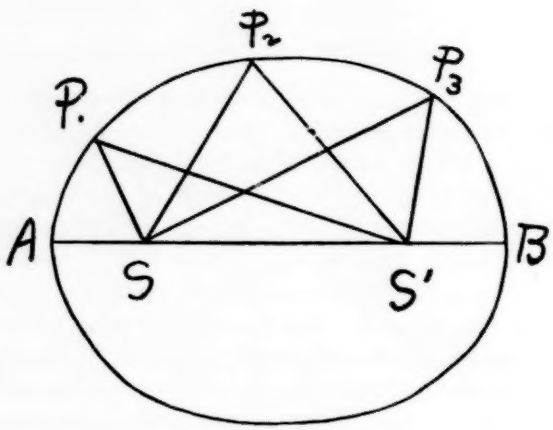
By W. P. WEBBER,
Louisiana State University.

It is quite generally known that Sir Isaac Newton discovered and published the law of nature known as the Law of Gravitation. Some may think that the law issued from his mind from sheer power of concentration as if by a supernatural gift of intelligence. The elementary text books do not give an account of the method by which Newton arrived at his conclusion. There is a legend to the effect that he was sitting in his garden meditating, when an apple fell from a tree and struck the ground. Newton is supposed to have reasoned some what as follows: I sit still on the earth. The apple falls to the ground while the earth does not seem to move. Suppose I sat still on the apple. Then it would not seem to move, but the earth would seem to fall to the apple. Now plausibly both may have fallen, each toward the other, until they met. Since the force acting between them apparently is mutual, and there being a great difference in the masses of the two bodies, it is fair to assume that the apple did most of the falling. The question of how much more the apple fell than the earth was still to be settled. We shall see below how this might have been done. As a matter of history, Newton was voted the discoverer of the law by the scientific society to which he belonged. But he, himself, said in a remark at the end of one of his scientific papers, on July 14, 1686, "The inverse square law holds in all celestial motions and was discovered also independently by my countrymen, Wren, Hooke and Halley."

Let us look into the back ground just a little. Newton lived from 1642 to 1721, while Kepler lived from 1571 to 1630. We know that Kepler discovered by actual measurement that the form of the earth's orbit about the sun is elliptical. It is common knowledge that the angle subtended at the eye by an object is inversely proportional to the distance of the object, provided that distance is quite large com-

pared to the size of the object. Kepler measured the angle subtended by the sun's diameter for all the days of the year.

From a fixed point representing the sun he drew to scale distances inversely proportional to these angles, and drew them at angles corresponding to the angle apparently passed over in the sky by the sun during the intervals between observations. He would obtain a figure similar to the one in the margin. This oval resembled an ellipse. It



was easy to prove it to be so. For the Greek mathematicians had, many centuries before, discovered all the important properties of the conic sections of which the ellipse is one type. Kepler had only to take the sums $SP_1+S'P_1$, $SP_2+S'P_2$, etc. and find that these sums are all equal to one another and to the length of the major diameter AB. This would prove, as far as measurement could prove, that the orbit of the earth is an ellipse.

Now Newton and his countrymen had knowledge of Kepler's laws. One of these laws was: "The radius vector (Line from the sun to the earth) sweeps over equal areas in equal intervals of time". That is if the planet moves from P_1 to P_2 in the same time that it moves from P_2 to P_3 then the areas of the two triangular figures SP_1P_2 and SP_2P_3 are equal. This fact could have been determined by measurement or by calculation. This law is known as the "Law of equal areas". Now we come to a case where mathematical research was needed to complete the work that Kepler had started. This appeared in the invention of the calculus by Newton himself. It was also

discovered in a somewhat different form, at about the same time by Leibnitz on the continent. After the knowledge of calculus became known among scholars, they attacked all sorts of problems by the new and powerful method. Some began to formulate mathematical theories of the planetary motions and mechanics in general. The instrument was now at hand by which the converse of Kepler's law of elliptic orbits could be solved. This instrument was the calculus. It could now be proved that if the elliptic orbit law and the equal area laws were true, the planet must be guided by a force through the sun which varies inversely as the square of the planet's distance from the sun. The inverse square law was necessary and sufficient along with the law of areas to explain scientifically the motions of the planets and the stars. Even today only slight corrections have been necessary. Anyone who cares to read up the mathematical treatment of the problem will find the simple fundamentals in any text on analytic mechanics. It may be said that Newton's differential equations of motion are essentially the ones still found in the books.

TO MISSISSIPPI MATHEMATICS TEACHERS

This word is addressed to my fellow mathematics teachers of Mississippi. By directing your attention to the Mathematics News Letter we hope to enlist your further cooperation in its circulation among our high schools and colleges. You will surely give us your best both for your own interests and that of your students.

Some years ago the college teachers of mathematics from Mississippi and Louisiana organized a section of the Mathematical Association of America. Later the National Council of Mathematics Teachers organized a similar section. The two Sections now meet in annual joint session. The Mathematics News Letter is the official journal of these sections. We cannot continue to publish it without the cooperation of our schools and colleges. A copy of the Letter should be on the library table of each high school, junior college, and senior college, for the use of students. Each teacher of mathematics should have a copy. We all talk about motivating and interesting our students in mathematics programs and we find that the journals now published are too technical for them to read. The News Letter provides an answer to the problem in the following very specific manner.

The teachers and college students write much of the

material for the News Letter. Herein is the opportunity for the teacher to lead discussions that are suitable for students to read. A variety of discussions covering topics in arithmetic, algebra, geometry, and the college subjects have appeared regularly during the past year. Any paper of interest to teachers is considered for publication. We can make the Letter suitable for reading and discussion in our high school and college classes. Cannot you see the opportunity here to use the News Letter in your classes and clubs for the most desirable end of cultivating the mathematical spirit?

Let me close with the specific request that you ask your school to subscribe for a library copy and that you subscribe for a personal copy so that your students may have guidance in reading and discussing the questions that interest them from time to time.

C. D. SMITH,
Head of Mathematics Department,
Mississippi A. & M. College.

A PLEA FOR MATHEMATICS

By VIOLET S. YOUNG
Baton Rouge High School

The advent of the twentieth century saw the beginning of a "world-wide movement in the teaching of mathematics". (1) The outcome of this movement was a wide-spread reform in the teaching of mathematics and an elaborate reorganization of the mathematics curriculum. Arithmetic still maintained its time-honored place in elementary school education, and mathematics—at least a certain rather large amount of mathematics—came to occupy a prominent part in any and all secondary school and college courses. And then something happened. Certain questions again demanded attention and answering, questions of "disciplinary values", "transfer of training", "practical values", "motivation", etc. There was much doubt in and investigation of the validity of the claim that training in one subject influenced performance in others. There was a sudden realization that in order to secure economy, effectiveness, and adequate return for time and effort spent there must be present in the learning process an active interest on the part of the learner. Along with this realization came a confusion of ideas; interest on the part of the learner became synonymous, in the minds of some, with a certain emotional condition in which pleasure was paramount. Certain

educators who failed to see that interest "frequently arouses in the individual willing devotion to toil and hardship and sometimes to experiences which are in themselves the reverse of pleasurable"[2] and who at the same time held that special qualities of mind, possessed by relatively few students, were requisite for success in mathematics judged this very necessary element of interest lacking in the mathematical endeavors of a large number of students. At the same time claims were made by others—just claims, perhaps—that beyond the merest elements of arithmetic mathematics is of little practical use to the average citizen, and to the large majority of people its value, though possibly great, is indirect. And so began the to and fro swing of the pendulum; again curricula were modified. To meet the demands of popular education mathematics requirements—in some instances even in courses that require more than elementary knowledge of chemistry, physics, and biology—have been much reduced or even entirely removed.

This paper makes no attempt to answer those ever-recurring questions referred to above, old questions that have taxed the ingenuity of educators as far back, at least, as Plato; it is just a plea on behalf of science teachers; a plea that the position of mathematics in the science curriculum be made secure.

That an intimate relation exists between science and mathematics needs no re-statement. It might be interesting, however, to read again these words of some of the masters:[3]

"Let no one who is unacquainted with geometry enter here."—*Plato.* (Said to have been over the entrance of his school of philosophy, the Academy.)

"God geometrizes continually."—*Plato.*

"Mathematics, the queen of the sciences."—*Gauss.*

"Mathematics is the glory of the human mind."—*Leibnitz.*

"Mathematics is the most marvelous instrument created by the genius of man for the discovery of truth."—*Laisant.*

"Mathematics is the indispensable instrument of all physical research."—*Berthelot.*

"All my physics is nothing else than geometry."—*Descartes.*

"There is nothing so prolific in utilities as abstractions."—*Faraday.*

"The two eyes of exact science are mathematics and logic."—*De Morgan.*

"All scientific education which does not commence with mathematics is, of necessity, defective at its foundation."—*Compte.*

"It is in mathematics we ought to learn the general method always followed by the human mind in its positive researches."—*Compte.*

"A natural science is a science only in so far as it is mathematical." —*Kant.*

"If the Greeks had not cultivated conic sections, Kepler could not have superseded Ptolemy."—*Whewell.*

Recently a study of some of the features affecting the articulation of high school and college chemistry was made by the writer. The results of this study indicated a marked agreement between degree of success in high school mathematics and degree of success in college chemistry. A brief summary of the study follows.

The entire high school and freshman college records of two hundred students, selected to secure representation of the city high school the small town high school and the consolidated country high school, were studied.

The students were divided into quartiles. The basis of this division was high school grades in chemistry and mathematics and general high school average. The students were then placed in three groups, namely, upper, middle, and lower. In each instance the upper group contained the first quartile, the middle group the middle half, and the lower group the fourth quartile. The results of tabulation of data were as follows:

University Grades	% MAKING A						% MAKING B						% MAKING C						% MAKING D						% MAKING E & F					
Groups	*Upper*		*Middle*		*Lower*		*Upper*		*Middle*		*Lower*		*Upper*		*Middle*		*Lower*		*Upper*		*Middle*		*Lower*		*Upper*		*Middle*		*Lower*	
Terms	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II	I	II
Grouping based on general high school record.	36						33						23						3.3						3.3					
		33						33						11						11						11				
			9.8						15						32						25						17			
				4.5						15						15						24						40		
					—						2.9						23						33						40	
						—						—						29						12						58
Grouping based on high school mathematical record.	36						30						17						2.8						14					
		33						13						20						27						1.6				
			7.6						13						31						32						15			
				8						14						24						18						36		
					—						6.1						30						24						38	
						—						—						20						24						56
Grouping based on high school chemistry record.	37						33						20						8.4						—					
		42						21						7.2						21						7.2				
			10						18						32						24						13			
				2.7						14						20						20						43		
					—						5.6						31						37						25	
						3.8						7.7						19						15						53

These results indicate that standing in high school mathematics is just as significant regarding standing in university chemistry as is high school general standing or even high school chemistry standing. A further interesting fact revealed by this study was that 51 percent of all chemistry failures in the university came from one college and that college, in spite of the fact that the nature of its work is fundamentally scientific requires *no* mathematics.

And finally, we may sustain our plea with excerpts from the "Aims of Mathematical Instruction" as given in a report on "a comprehensive study of the whole problem concerned with the improvement of mathematical education" conducted by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America.

"A progressive increase in the pupil's understanding of the nature of the fundamental operations and power to apply them in new situations.

Exercise of common sense and judgment in computing from approximate data, familiarity with the effect of small errors in measurements, the determination of the number of figures to be used in computing and to be retained in the result, and the like.

The development of self-reliance in the handling of numerical problems through the consistent use of checks on all numerical work.

Appreciation of the significance of formulas and ability to work out simple problems by setting up and solving the necessary equations.

Ability to understand and interpret correctly graphic representations of various kinds . . . . as well as understanding of various sorts of dependence of one variable quantity upon another.

Familiarity with the *geometric forms* common in nature, industry, and life; the elementary properties and relations of these forms, including their *mensuration*; the development of *space-perception*; and the exercise of *spatial imagination*

The acquisition, in precise form, of those ideas or concepts in terms of which the quantitative thinking of the world is done.

The ability to think clearly in terms of such concepts and ideas. This ability involves training in—

Analysis of a complex situation into simpler parts. This includes the recognition of essential factors and the rejection of the irrelevant.

Recognition of logical relations between inter-dependent factors

and the understanding and, if possible, the expression of such relations in precise form.

Generalization; that is, the discovery and formulation of general laws and an understanding of their properties and applications.

The acquisition of mental habits and attitudes which will make the above training effective in the life of the individual. Among such habitual reactions are the following: a seeking for relations and their precise expression; an attitude of inquiry; a desire to understand, to get to the bottom of a situation; concentration and persistence; a love for precision, accuracy, thoroughness, and clearness, and a distaste for vagueness and incompleteness; a desire for orderly and logical organization as an aid to understanding and memory."

If mathematics can lay claim to the achievement of even a small part of these ends, certainly there can be none who will speak against this plea: in courses that are in any degree—even ever so small a degree—scientific in nature let the mathematics requirements be made full as to content and exacting as to accomplishment on the part of the student.

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ON THE FIRST TWO CHAPTERS OF RUSSELL'S INTRODUCTION TO MATHEMATICAL PHILOSOPHY

By S. T. SANDERS

In his preface to "Introduction to Mathematical Philosophy", Bertrand Russell notes a possible (probable) distinction between mathematical philosophy and the philosophy of mathematics by implying that one is philosophy and the other is mathematics. Yet, strangely enough he adopts the former phrase, not the latter, in naming his book. The Editor, in a note following the preface, while indicating disagreement with Russell in his use of the distinction, seems to go with him a part of the way which he had pointed out in the words, "It is to be hoped that some readers may be sufficiently interested to advance to a study of the method by which mathematical logic can be made helpful in investigating the traditional problems of philosophy."

In Chapter I which he calls "The Series of Natural Numbers" his instinct for regarding the logic rather than the technical aspect of mathematics is shown in the first paragraph where he says, "Mathematics is a study which, when we start from its most familiar portions may be pursued in either of two opposite directions". Then he continues, "The more familiar direction is constructive, towards gradually increasing complexity: from integers to fractions, real numbers, complex numbers; from addition and multiplication to differentiation and integration, and on to higher mathematics. The other direction, which is less familiar, proceeds, by analyzing, to greater and greater abstractness and logical simplicity; instead of asking what can be defined and deduced from what is assumed to begin with, we ask instead what more general ideas and principles can be found, in terms of which what was our starting point can be defined, or deduced."

Such an orientation of the subject quite naturally led to his fixing upon a starting point for proper technical mathematics. This he identifies with the series 0, 1, 2, 3, ... n, n+1, . . .

He throws a light upon the teaching of mathematics, a light disclosing most subtle difficulties, when he remarks "It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number 2: the degree of abstraction involved is far from easy. And the discovery that 1 is a number must have been difficult. As for 0, it is a very recent addition;

the Greeks and Romans had no such digit." Thus, the light thrown upon teaching is that of philosophy.

While conceding that an endless sequence of definitions, one in terms of its predecessor, is an absurdity, a minimum number of independent primitive ideas may consistently be made an object of search. Thus, on page 5 he says, "Having reduced all traditional pure mathematics to the theory of the natural numbers, the next step in logical analysis was to reduce this theory itself to the smallest set of premises and undefined terms from which it could be derived. This work was done by Peano."

It is a remarkable tribute paid to Peano by Russell when he says "The work of analyzing mathematics is extraordinarily facilitated by this work of Peano."

Peano hangs all arithmetic upon three primitive ideas, namely,

0, NUMBER, SUCCESSOR.

He arbitrarily adopts five so-called primitive propositions so chosen that they

(a) result in well-defined properties for the three primitive ideas,
(b) imply consistency in these properties and, furthermore,
(c) imply a complete theory of numbers.

The following are the five primitive propositions:

(1) 0 is a number.
(2) The successor of any number is a number.
(3) No two numbers have the same successor.
(4) 0 is not the successor of any number.
(5) Any property which belongs to 0 and to the successor of any number that has the property, belongs to all numbers.

Passing over Russell's remarks and illustrations of the effectiveness of Peano's three primitive ideas and five primitive propositions as a logical basis of the arithmetic of the natural numbers, we find in the closing pages of Chapter I a digest of the considerations which require one to pass from Peano's "analysis of arithmetic" to the logical background or setting of his five primitive propositions and his three primitive ideas, if a true approach to the philosophy of mathematics is to be made. Thus arrives Chapter II in which a definition of NUMBER is set forward.

After crediting Frege with having given the first correct answer to the question, "What is a number?", he makes the statement, al-

most startling to one who reads it for the first time, that Frege's definition, though of the utmost importance, (it was made in 1884) "attracted almost no attention." . . "until it was rediscovered by himself (Russell) in 1901".

The genius of the man for making clear expression of ideas within themselves abstract and elusive is shown nowhere more satisfyingly than in the second paragraph of Chapter II. We quote it in full. "Many philosophers, when attempting to define number, are really setting to work to define plurality, which is quite a different thing. Number is what is characteristic of numbers, as man is what is characteristic of men. A plurality is not an instance of number, but of some particular number. A trio of men, for example, is an instance of the number 3, as 3 is an instance of number. This point may seem elementary and scarcely worth mentioning; yet it has proved too subtle for the philosophers, with few exceptions."

While we are not prepared to cite all instances which furnish point to Russell's statement that the philosopher often uses an inadequate definition of number, it does occur to us to mention Bergson as one. In Chapter II of his "Time and Free Will" (in which he makes somewhat extended remark on the nature of number) he says, "Number may be defined as a collection of units". Not only does the definition imply mere plurality, a limitation in keeping with Russell's criticism, but even that notion is but sloppily implied in the term "collection". Think of collecting an infinite quantity of units!

Essentially, the Frege-Russell conception of number is grounded in the notion of a one-to-one correspondence between classes of things. Two classes of things are SIMILAR if the members of one class can be placed in one-one correspondence with the members of the other class. Thus, the set of ALL similar classes, is itself a class, which is called the NUMBER of each class in the set. When we say there are three apples in the basket we mean that "three" is the number class to which the apples belong.

As will be shown later, in the Russell treatise, the outstanding advantage of this definition of NUMBER is that it carries with it the beginnings of a theory of infinite classes, i. e., classes with an infinite number of members in it. In other words, such a conception of number is perfectly adapted to guarantee adequate mathematical counterpart to such physical conceptions as a material universe without bounds, or an unlimited extension of matter.

A one-one relation between two classes of things can be determined by enumeration only if the number of members of each class is finite. But, whether the class be infinite of finite, counting is not necessary to establish whether or not it is similar to another class, since if the two classes are infinite, counting will not avail, and if the two are finite, the testing for one-one correspondence between the two sets of members need not involve counting.

PROBLEM DEPARTMENT

Edited by T. A. BICKERSTAFF
University of Mississippi

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

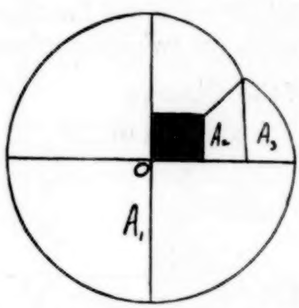
Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 5. Proposed by R. M. Guess, University, Miss.
Solved by Jessie May Hoag, Jennings, La.

If a cow is tied with a 75 foot rope to the corner of a barn 25 feet square, over how much territory can she graze?



On two sides of the barn the cow can graze in a circle of radius 75′. The area is:

$$A_1 = \tfrac{3}{4}\pi(75)^2$$

The second and fourth corners become centers of circles of radius 50 completing the boundary.

These circles, $x^2+(y-25)^2=50^2$

and $(x-25)^2+y^2=50^2$

intersect at (45.56, 45.56).

$$A_2 = \int_{25}^{45.56} x\,dx = \frac{x^2}{2}\Big]_{25}^{45.56} = 72.56$$

$$A_3 = \int_{45.56}^{75} \sqrt{50^2 - (x-25)^2}\,dx$$

$$= \left[(x-25)\sqrt{50^2-(x-25)^2} + 50^2 \arcsin \frac{x-25}{50} \right]_{45.56}^{75}$$

The entire grazing area $= A_1 + 2A_2 + 2A_3 -$
This is about 16,633.8 square feet.

No. 3. Proposed by L. E. Scally, Morse, La.
Solved by Jessie May Hoag, Jennings, La.

To construct an athletic track one-fourth mile long in the form of an ellipse with major axis equal twice the minor axis.

Let the equation of the ellipse be

$$\frac{x^2}{4b^2} + \frac{y^2}{b^2} = 1$$

$$1320 = 4\int_0^{2b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= 2\int_0^{2b} \sqrt{\frac{16b^2-3x^2}{4b^2-\ x^2}}$$

$$= 8b \int_0^{\frac{\pi}{2}} \sqrt{1 - \tfrac{3}{4}\sin^2\theta}\; d\theta,$$

Where $x = 2b \sin \theta$,

$$1320 = 8b \left| \frac{\pi}{2} \left(1 - \tfrac{3}{16} - \frac{9}{128} - \frac{27}{1024} \quad . \; . \; . \; . \right) \right|$$

$$b = \frac{1320}{8. \frac{\pi}{2}. .7} \text{ approximately}$$

$$= 150 \text{ ft. approximately}$$

Therefore the axes of the ellipse are approximately 300 and 600 ft. long. The foci are at $(\pm\ 150\sqrt{3}, 0)$ and the sum of the distances from any point on the ellipse to the foci is 600 ft.

To construct the ellipse, measure from the center 258.8 ft. in each direction along the major axis. Stake the ends of a 600 ft. tape to these points and pull the line taut. The vertex of the angle formed is a point on the ellipse.

PROBLEMS FOR SOLUTION

Proposed by the Editor

No. 7. A bullet strikes the face of a wooden beam with a velocity of 1000 ft. per second. It is brought to rest at the opposite face after passing through 6 inches of wood after 1/24 second. Find its velocity at the instant when it had passed through 1 inch of wood if the retardidation is constant.

To all Teachers of Mathematics in the South:

The American Mathematical Society, and the Mathematical Association of America hold their annual meetings in New Orleans from December 28 to December 31, 1931. The meetings will be in the Administration Building of Sophie Newcomb College. The Department of Mathematics at Tulane University, of which Sophie Newcomb is a part, urges the teachers of the South to attend these meetings. The Society has never come to the South for its meetings before. It therefore should be a matter of pride with the Southern teachers to attend.

Mathematics is a difficult subject. It cannot be understood or appreciated without long years of study. Therefore, the public at large is often unsympathetic, and sometimes even hostile towards our attempts to build up a sound scholarship. It is particularly necessary that the teachers of Mathematics gain their inspiration from associations with each other. Enthusiasm is easily aroused when you once see Mathematicians working together.

The Society and the Association exist because those whose vocations involve knowledge and use of Mathematics in its various forms can serve most effectively through the acquaintance and cooperation with others of related interests.

Therefore every man and woman who is directly or indirectly interested in the service rendered to mankind by the science of Mathematics is invited to become a member of either the Society, or Association, or both. It welcomes equally the investigator whose interest is to extend the bounds of Mathematics, and the teacher whose function is to expound the most powerful and most accurate methods now known to the coming generation.

Information concerning the meetings at New Orleans may be secured from any one of the undersigned, who have been appointed by the Society as a Committee to conduct the meetings at New Orleans.

H. E. BUCHANAN, *Chairman*, Tulane.
G. C. EVANS, Rice Institute, Houston.
W. L. MISER, Vanderbilt.
W. P. WEBBER, L. S. U.